LOYOLA COLLEGE (AUTONOMOUS) CHENNAI 600 034

B. Sc. Degree Examination – Mathematics

Fifth Semester – November 2014

MT 5406 – COMBINATORICS

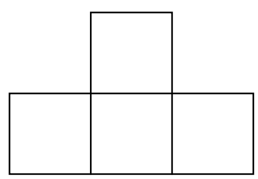
Date: Max: 100 Marks

Time: Dept. No.

SECTION A

ANSWER ALL QUESTIONS. **(10 × 2 = 20)**

1. How many 4-letter words with distinct letters can be got from the word UNIVERSAL?
2. Define Bell number.
3. Find the ordinary generating function of 5 symbols a, b, c, d, e.
4. Define binomial number.
5. Define permanent of a matrix.
6. State generalized inclusion and exclusion principle.
7. Find the rook polynomial for the chess board C given below.



8. Evaluate $\varphi(720)$.
9. Explain the term cycle index.
10. Define G-equivalence between two sets.

SECTION B

ANSWER ANY FOUR QUESTIONS. **(5 × 8 = 40)**

11. There are 30 females, 35 males in a junior class while there are 25 females and 20 males in a senior class. In how many ways can a committee of 10 be chosen so that there are exactly 5 females and 3 juniors in the committee?
12. (i) Define exponential generating function.
    (ii) Find the number of r letter sequences that can be formed using the letters P, Q, R and S such that in each sequence there are an odd number of P's and an even number of Q's. (2 + 6)
13. Prove that the element f of $R[t]$ given by $f(t) = \sum_{k=0}^{\infty} a_k t^k$ has an inverse in $R[t]$ if and only if a_0 has an inverse in R.
14. Derive the formula to find the sum of squares of first n natural numbers.

15. How many integers between 1 and 300 are (i) divisible by at least one of 3, 5, 7 (ii) divisible by 3 and 5 but not by 7 (iii) divisible by 5 but reither by 3 nor 7?
16. Show that the number of derangements of set with n objects is $D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ ... + (-1)^n \frac{1}{n!}\right]$ elements to a set with n elements.
17. Show that 97 is the 25^{th} prime number.
18. Find the number of 6-letter words that can be formed using the letters A, B and C (each twice) in such a manner that A does not appear in the first 2 positions, B does not appear in the third position and C does not appear in the fourth and fifth positions.

SECTION C

ANSWER ANY TWO QUESTIONS. (2 x 20 = 40)

19. a) Prove that the number of distributions of n distinct objects into m distinct boxes with the objects in each box arranged in a definite order is $[m]^n$.
    b) Find the ordinary generating function of the sequence $\{(r+n-1)C_{n-1}\}_{r \geq 0}$ by differentiation of infinite geometric series. (10 + 10)

20. a) Define Stirling numbers of second kind. Formulate a table for S_n^m, for $1 \leq m, n \leq 6$.
    b) State and prove Multinomial theorem. (10 + 10)

21. State and solve Menage problem.

22. State and prove Burnside's Lemma.

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